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ON ε −PREOPEN SETS

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Abstract. In general topology, the word "preopen" plays a major role and hence, the concept of the "preopen compactness" has been developed. In this paper, the focal point is to establish few theorems based on ε-preopen compactness. There are also numerous associations with other forms of compactness. Furthermore, new axioms for separation are described.

Keywords :$\varepsilon -$ preopen set, $\varepsilon -$ preopen compact- ness.

Introduction

Throughout the article, $(\mathcal{S},\ \varphi)$ represents a non-empty topological space in this text Space on which, unless otherwise stated, no separation axioms are presumed.The closure and interior of $\mathcal{M} \subset \mathcal{S}$ are denoted by $Cl(\mathcal{M})$ and $Int(\mathcal{M})$ respectively.Monsef has presented essential concepts of pretopologicalconcepts[16], while preopen concepts of theory can be examined in Andrijevic's concepts[17]. The concept of preopen sets was introduced and investigated by Mashhour et al. [15].$(\mathcal{S},\ \varphi)$ is called s −closed, if every semi-open cover has a finite subfamily the semi-closures of whose members cover $\mathcal{S}$.A subset $\mathcal{M}$ of topological space $(\mathcal{S},\ \varphi)$ is said to be preopen [15] if $\mathcal{M} \subset Int(Cl(\mathcal{M}))$ holds. We denote by $PO(\mathcal{S},\ \varphi)$ (sometimes, $P\ O(X)$) the set of all preopen sets in $(\mathcal{S},\ \varphi)$ [16]. The complement of a preopen set is called preclosed. The intersection of all preclosed sets of $(\mathcal{S},\ \varphi)$ containing a subset $\mathcal{M}$ is called the preclosure of $\mathcal{M}$ and is denoted by $pCl(\mathcal{M})$ [5]. The union of all preopen sets contained in a subset $\mathcal{M}$ is called the preinterior of $\mathcal{M}$ and is denoted by $pInt(\mathcal{M})$. The set $pCl(\mathcal{M})$ is preclosed and $pInt(\mathcal{M})$ is preopen in $(\mathcal{S},\ \varphi)$ for any subset $\mathcal{M}$ of $(\mathcal{S},\ \varphi)$, because an arbitrary union of preopen sets of $(\mathcal{S},\ \varphi)$ is preopen [1]. It is well known that [2, Theorem 1.5 (e)(f)] $pCl(\mathcal{M}) = \mathcal{M} \cup Cl(Int(\mathcal{M}))$ and $pInt(\mathcal{M}) = \mathcal{M} \cap Int(Cl(\mathcal{M}))$ hold for any subset $\mathcal{M}$ of $(\mathcal{S},\ \varphi)$. We note that $\tau \subset PO(\mathcal{S},\tau)$ for any topological space $(\mathcal{S}, \varphi)$and $PO(\mathcal{S},\tau)$is not a topology on $\mathcal{S}$in general. Also, A **function** $f : (X,\tau) \to (Y,\sigma)$ is said to be completely **irresolute** [10] if $f^{-1}(V)$ is regular open in $(\mathcal{S},\tau)$ for every semi-open set Vin Y.

Preliminaries and Main Results

Definition 1. A space $(\mathcal{S}, \varphi)$ is ε −po-compact if every ε −po-cover (a cover consisting of ε −po- sets) of $\mathcal{S}$ has a finite subcover.

Equivalently, $(\mathcal{S}, \varphi)$ is ε −po-compact if every ε −po-cover of $\mathcal{S}$ has a finite subcover. A submaximal space is an example of a ε −po-compact space. The proof of the following result follows from the fact that every open set is a ε −po set.

Theorem 1. If $(\mathcal{S}, \varphi)$ is a ε −po-compact space, then it is s −closed.

Proof. Let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be a semi-open cover of $\mathcal{S}$. Then $\mathcal{M}$ is a ε −po-cover of $\mathcal{S}$.Since$\mathcal{S}$ is ε −po-compact, it has a finite subcover such that

$\mathcal{S} \subseteq \bigcup_{i=1}^{n} \mathcal{M}_{\alpha i}$.But $\bigcup_{i=1}^{n} \mathcal{M}_{\alpha i} \subseteq \bigcup_{i=1}^{n} scl(\mathcal{M}_{\alpha i})$, so $\mathcal{S}$is s −closed.

Since an$s$ −closed space is S −closed, a ε −po-compact space is s −closed.

Theorem 2. If a map $f : (\mathcal{S}, \varphi) \to (Y,\sigma)$ is ε-po-irresolute (resp., ε −po- continuous) surjective and K is ε −po-compact subset of $\mathcal{S}$, then $f(K)$ is ε −po-compact (resp, QHC) in Y.

Proof. Let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be any ε −po- (open) cover of $f(K)$. Since f is ε −po-irresolute (resp, ε-po-continuous), $\mathcal{M}* = \{f^{-1}(\mathcal{M}_\alpha) : \alpha \in \Delta\}$ is ε −po- cover of K where K is ε −po-compact. Thus it has a finite subcover. That is $\mathcal{S} \subseteq \bigcup_{i=1}^{n} f^{-1}(\mathcal{M}_{\alpha i})$. Since, f is ε −po-irresolute (ε-po-continuous) and surjective, we have $f(K) \subseteq \bigcup_{i=1}^{n} f(f^{-1}(\mathcal{M}_{\alpha i})) \subseteq \bigcup_{i=1}^{n}(\mathcal{M}_{\alpha i})$ $(resp., f(K) \subseteq \bigcup_{i=1}^{n}(\mathcal{M}_{\alpha i}) \subseteq \bigcup_{i=1}^{n} cl(\mathcal{M}_{\alpha i}))$. Thus $f(K)$ is ε −po-compact (resp., QHC) in Y .

Definition 2. A space $(\mathcal{S},\ \varphi)$ is strongly s −compact if for every ε −po-cover $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$, there exist $\mathcal{M}_{\alpha 1}, \mathcal{M}_{\alpha 2}, \dots, \mathcal{M}_{\alpha n} \in \mathcal{M}$ such that $\mathcal{S} \subseteq \bigcup_{i=1}^{n} cl(\mathcal{M}_{\alpha i})$.

If $(\mathcal{S},\ \varphi)$ is ε −po-compact, then clearly it is strongly ε −po-compact, since $\mathcal{M}_{\alpha i} \subseteq \bigcup_{i=1}^{n} cl(\mathcal{M}_{\alpha i})$for every $\mathcal{M}_{\alpha i} \subseteq \mathcal{S}, i = 1, 2\dots, n$.

Definition 3. A space $(\mathcal{S},\ \varphi)$ is strongly O−ε −po-regular if $\mathcal{S}$ has a ε −po-cover $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ for all $x \in \mathcal{S}$, and for all $\mathcal{M}_\alpha(x) \in \mathcal{M}$ such that $x \in \mathcal{M}_\alpha(x)$, there is an existence of $\wp_x \in \varepsilon - po(\mathcal{S})$ such that $x \in \wp_x \subseteq cl(\wp_x) \subseteq \mathcal{M}_\alpha(x)$.

Theorem 3. If $(\mathcal{S},\ \varphi)$ is strongly ε-po-compact and strongly O−ε −po-regular, then it is ε −po-compact.

Proof. Let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be a ε −po-cover of $\mathcal{S}$. Then since $\mathcal{S}$ is strongly O−ε −po-regular, for all x $\in\mathcal{S}$, there is an existence of $\mathcal{M}_\alpha(x) \in \mathcal{M}$ such that $x \in \mathcal{M}_\alpha(x)$ and there is an existence of $\wp_x \in$ ε-$po(\mathcal{S})$ with $x \in \wp_x \subseteq cl(\wp_x) \subseteq \mathcal{M}_\alpha(x)$. Thus $\{\wp_x : x \in \mathcal{S}\}$ is a ε −po-cover of $\mathcal{S}$. Since $\mathcal{S}$ is strongly ε −po-compact, there exist $\wp_{x2}, \wp_{x2}, \dots, \wp_{xn}$ such that $\mathcal{S} \subseteq \bigcup_{i=1}^{n} cl(\wp_{xi}) \subseteq \mathcal{M}_{\alpha i}(x)$. Therefore $\mathcal{S}$ is ε −po-compact.

Corollary 1. If $(\mathcal{S},\ \varphi)$ is strongly $O - \varepsilon$ −po-regular, then it is strongly ε −po- compact if and only if it is ε −po-compact.

Definition 4. A space $(\mathcal{S},\ \varphi)$ is ε −po-regular if for every ε −po-cover $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$of $\mathcal{S}$ and for every $x \in \mathcal{M}_\alpha(x) \in \mathcal{M}$, there is an existence of a preopen set $\wp_\alpha$ such that $x \in \wp_\alpha \subseteq \mathcal{M}_\alpha$.

Theorem 5. If $(\mathcal{S}, \tau)$ is a ε-po-regular and a strongly compact space, then it is ε −po-compact.

Proof. Let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be a ε −po-cover of $\mathcal{S}$. Then since $\mathcal{S}$ is ε-po-regular, for all $x \in \mathcal{M}_\alpha(x)$ there is an existence of $\wp \in po(\mathcal{S})$ such that $x \in \wp_\alpha \subseteq$

$\mathcal{M}_\alpha$ for all α ∈ Δ and so $\{\wp_\alpha : \alpha \in \Delta\}$ is a preopen cover of $\boldsymbol{\mathcal{S}}$. Since $\boldsymbol{\mathcal{S}}$ is strongly compact, then $\boldsymbol{\mathcal{S}} \subseteq \bigcup_{i=1}^{n} \wp_{\alpha i} \subseteq \bigcup_{i=1}^{n} \mathcal{M}_{\alpha i}$. Therefore $\boldsymbol{\mathcal{S}}$ is ε −po-compact.

The converse of the preceding Theorem need not be true since a pre- open set need not be a ε −po- set. Moreover, it is clear that the notions of ε −po-regular and ε −po-irresolvable spaces are independent. The proof of the following result follows immediately from Theorem 8 and Theorem 9.

Corollary 2. If a space $(\boldsymbol{\mathcal{S}}, \varphi)$ is ε −po-regular and ε −po-irresolvable, then it is ε −po-compact if and only if it is strongly compact.

Definition 5. A space $(\boldsymbol{\mathcal{S}}, \varphi)$ is o-regular if for every ε −po-cover $\mathcal{M} = \{\mathcal{M}_{\alpha i} : \alpha \in \Delta\}$ of $\boldsymbol{\mathcal{S}}$ and for every $x \in \mathcal{M}_\alpha(x) \in \mathcal{M}$, there is an existence of an open set $\wp$, such that $x \in \wp_x \subseteq cl(\wp_x) \in \mathcal{M}_\alpha(x)$.

Theorem 6. If $(\boldsymbol{\mathcal{S}}, \tau)$ is o-regular, then it is ε −po-compact if and only if it is QHC.

Proof. Let $(\boldsymbol{\mathcal{S}}, \varphi)$ be ε −po-compact. By Theorem 7, $(\boldsymbol{\mathcal{S}}, \varphi)$ is a compact space, so it is QHC −space.

Conversely, let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be a ε −po- cover of $\boldsymbol{\mathcal{S}}$. Since $\boldsymbol{\mathcal{S}}$ is o −regular, for every $x \in \mathcal{M}_\alpha(x) \in \mathcal{M}$, there is an existence of an open set U such that $x \in \wp_x \subseteq cl(\wp_x) \subseteq \mathcal{M}_\alpha(x)$. Thus $\{\wp\alpha : \alpha \in \Delta\}$ is an open cover of $\boldsymbol{\mathcal{S}}$. Since $\boldsymbol{\mathcal{S}}$ is QHC, there is an existence of a finite family such that $\boldsymbol{\mathcal{S}} \in \bigcup_{i=1}^{n} cl(\wp_{xi}) \subseteq \bigcup_{i=1}^{n} \mathcal{M}_{\alpha i}$. Hence (X, φ) is ε −po-compact.

Corollary 3. If $(\boldsymbol{\mathcal{S}}, \varphi)$ is o-regular, then it is ε −po-compact if and only if it is s −closed (S −closed, compact, nearly compact).

Proof : Clearly s −closed (S −closed, compact, nearly compact) imply QHC and by Theorem 10, QHC imply ε −po-compact.

Conversely, if $(\boldsymbol{\mathcal{S}}, \tau)$ is ε −po-compact, then by Theorem 7 it is compact and so it is nearly compact and by Theorem 12, it is s −closed. Thus it is S −closed.

Theorem 7. If $(\boldsymbol{\mathcal{S}}, \tau)$ is o −regular, then it is QHC if and only if it is strongly ε −po-compact.

Proof. Let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be a ε −po-cover of $\boldsymbol{\mathcal{S}}$. Since $\boldsymbol{\mathcal{S}}$is o- regular, for each and every $x \in \mathcal{M}_\alpha(x) \in \mathcal{M}$ there is an existence of an open set $\wp$ such that $x \in \wp_x \subseteq cl(\wp_x) \subseteq \mathcal{M}_\alpha(x)$. So $\{\wp_\alpha : \alpha \in \Delta\}$is an open cover of $\boldsymbol{\mathcal{S}}$. Since $\boldsymbol{\mathcal{S}}$is QHC, $\boldsymbol{\mathcal{S}} \subseteq \bigcup_{i=1}^{n} cl(\wp_x) \subseteq \bigcup_{i=1}^{n} \mathcal{M}_{\alpha i}(x)$. Since $\bigcup_{i=1}^{n} \mathcal{M}_{\alpha i} \subseteq \bigcup_{i=1}^{n} \mathcal{M}_{\alpha i}$for every $\mathcal{M}_{\alpha i} \subseteq \boldsymbol{\mathcal{S}}, i = 1,2\ldots,n.,\boldsymbol{\mathcal{S}}$ is strongly ε-po- compact. Conversely, let $\mathcal{M} = \{\mathcal{M}_\alpha : \alpha \in \Delta\}$ be an open cover of $\boldsymbol{\mathcal{S}}$. Then $\mathcal{M}$ is a ε-po- cover of $\boldsymbol{\mathcal{S}}$ and since $\boldsymbol{\mathcal{S}}$ is strongly ε −po-compact, $\boldsymbol{\mathcal{S}} \subseteq \bigcup_{i=1}^{n} cl(\mathcal{M}_{\alpha i}(x))$. Thus $\boldsymbol{\mathcal{S}}$is QHC.

Separation axioms via ε −po-sets

In this section, several new separation axioms via ε −po- sets are introduced. Connections to other well-known ones are also discussed.

Definition 6. Let $(\boldsymbol{\mathcal{S}}, \varphi)$ be a topological space. Then

(i) $(\boldsymbol{\mathcal{S}}, \varphi)$ is called a T_{p0} – space if for each pair of distinct points $x, y \in \boldsymbol{\mathcal{S}}$, there is either a ε −po- set containing x but not y or a ε −po- set containing y but not x.

(ii) $(\boldsymbol{\mathcal{S}}, \varphi)$ is called a T_{p1} – space if for each pair of distinct points $x, y \in \boldsymbol{\mathcal{S}}$, there is a ε −po- set containing x but not y, and a ε −po- set containing y but not x.

(iii) $(\boldsymbol{\mathcal{S}}, \varphi)$ is called a T_{p2} – space if for each pair of distinct points x, $y \in \boldsymbol{\mathcal{S}}$, there exist ε −po- sets $\wp$ and V such that $x \in \wp, y \in V$ and $\wp \cap V = \varphi$.

(iv) $(\boldsymbol{\mathcal{S}}, \varphi)$is called a weak regular space if for each closed subset $F \subset \boldsymbol{\mathcal{S}}$ and each point x does not belongs to F, there exist ε −po- sets $\wp$ and V such that $x \in \wp, F \subset V$ and $\wp \cap V = \varphi$. A weak regular T_{p1} – Space is called T_{p3} – space.

(v) $(\boldsymbol{\mathcal{S}}, \varphi)$is called a weak normal space if for each pair of disjoint closed subsets F_1 and F_2 of $\boldsymbol{\mathcal{S}}$ there exist ε −po sets $\wp$ and V such that that $F_1 \subset \wp$, $F_2 \subset V$ and $\wp \cap V = \varphi$. A weak normal T_{p1} –

Space is called a T_{p4} − space.

It is clear that the T_{p2} condition implies the T_{p1} condition, which in turn implies the Tp∘ condition. Since the notions of open set and ε −po set are independent, weak regularity and regularity are also independent notions.

Theorem 12. If each point $x \in \mathcal{S}$ is a ε −po set, then the space $(\mathcal{S}, \varphi)$ is a T_{p1} − Space.

Proof. Let each point of $\mathcal{S}$ be a ε −po subset of $\mathcal{S}$. If $|\mathcal{S}| = 1$, the result is clear. Thus let x and y be distinct points of $\mathcal{S}$. Then $\{y\}$ is a ε −po set, so $\mathcal{S}\backslash\{y\}$ is a ε −po set containing x but not y. It follows that $(\mathcal{S}, \varphi)$ is a T_{p1} − Space.

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